MODEL OF GAS BUBBLE BREAKUP IN A TURBULENT LIQUID FLOW

V. A. Sosinovich, V. A. Tsyganov,
B. A. Kolovandin, B. I. Puris, and
V. A. Gertsovich

UDC 532.529.6

A system of equations for evolution of the size spectrum of gas bubbles as a result of their breakup in an isotropic turbulent damped flow of an incompressible liquid is derived and solved numerically.

Some technological processes, e.g., water purification and flotation of ores, require a gas-liquid medium with a known size distribution of gas bubbles and a prescribed density. One of the methods for preparing such a medium, adopted in practice, may be gas dispersion under hydrodynamic cavitation conditions followed by gas bubble breakup in a turbulent liquid flow [1]. The present work is an attempt to construct a theoretical model of gas bubble breakup in a turbulent flow. The theoretical model is formulated as a problem of calculation of the evolution of a prescribed probability density of gas bubble size distribution in an isotropic turbulent flow of an incompressible liquid. The initial form of the distribution is chosen such that it may be implemented under hydrodynamic cavitation conditions. As is known from experiments [1], in the range of cavitation numbers of $K \simeq 0.03-0.05$ the amount of gas absorbed by a liquid increases drastically. At the same time the characteristic size of gas bubbles is commensurable with the size of inhomogeneities of the cavities formed in this regime (of the order of several millimeters). Further evolution of the gas-liquid medium obtained in this way involves bubble breakup by turbulent velocity pulsations, bubble collision, and their coalescence. In the present work we will consider only turbulent breakup of gas bubbles without regard for coalescence. Owing to this restriction the gas saturation effect is insignificant. The influence of gas saturation and coalescence on the size spectrum of bubbles will be taken into account in subsequent work.

1. Distribution Function $f_t(r)$. As the subject of investigation, we will choose a probability density function $f_t(r)$ that is defined so that the expression $f_t(r)dr$ is the probability of finding a gas bubble in the size range $r-(r+dr)$ in a unit volume of a liquid. From the definition it follows that the normalization condition is

$$\int_0^\infty f_t(r)\,dr = 1\,, \tag{1}$$

which means that at any moment of time some bubble will certainly be found at any point of a homogeneous turbulent flow. A state of the complete absence of bubbles is described by the function $f_t(r) = \delta(r)$, where $\delta(r)$ is the Dirac function. The function $f_t(r) = \delta(r-L)$ corresponds to a state of the system with bubbles of the same size $r = L$.

The function $f_t(r)$ describes the portion of bubbles with size r:

$$f_t(r) = n_t(r)/N(t)\,, \tag{2}$$

where $n_t(r)$ is the quantity of bubbles with size r, and $N(t)$ is the total number of bubbles per unit volume of the gas-liquid system:

Academic Scientific Complex "A. V. Luikov Heat and Mass Transfer Institute, Academy of Sciences of Belarus," Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 68, No. 2, pp. 192-204, March-April, 1995. Original article submitted December 23, 1993.

 1062-0125/95/6802-0164$12.50 ©1996 Plenum Publishing Corporation

$$N(t) = \int_0^\infty n_t(r)\,dr. \tag{3}$$

It is apparent that in the course of breakup $n_t(r)$ and $N(t)$ change with time. The expression

$$V_g(r, t) = \frac{4}{3}\pi r^3 n_t(r) \tag{4}$$

determines the volume of the gas contained in bubbles with radius r. The expression

$$V_g(t) = \frac{4}{3}\pi \int_0^\infty r^3 n_t(r)\,dr \tag{5}$$

relates the total gas volume per unit volume of the gas-liquid system to the volume of the gas contained in bubbles with radius r. Using formulas (2) and (5), it is easy to obtain a relationship between $V_g(t)$ and the distribution function $f_t(r)$:

$$V_g(t) = \frac{4}{3}\pi N(t) \int_0^\infty r^3 f_t(r)\,dr. \tag{6}$$

In formulating the problem of turbulent breakup of gas bubbles the function $V_g(t)$ must be prescribed as the initial condition

$$V_g(t)\,|_{t=0} \equiv V_g(0). \tag{7}$$

In this case, we may write an expression for the total quantity of bubbles:

$$N(t) = V_g(0)/\overline{V}_g(t), \tag{8}$$

where

$$\overline{V}_g(t) = \frac{4}{3}\pi \int_0^\infty r^3 f_t(r)\,dr \tag{9}$$

is the mean volume of a bubble. Obviously, in a breakup process $N(t)$ is a function that increases monotonically with time.

2. Breakup Condition of Bubbles in a Turbulent Flow. Bubble breakup in a turbulent flow occurs when the intensity of turbulent velocity fluctuations exceeds some threshold value that is different for different bubble sizes. In other words, for a turbulent field of prescribed intensity some critical bubble size exists such that bubbles with a radius larger than the critical one break up.

We now write a formula for the critical bubble size in the form (formula (89.3) in [2])

$$a_{cr} \cong (1/k_f)^{1/3}\,\sigma\,(\rho/\rho')^{1/3}/(\rho U^2/2). \tag{10}$$

where k_f is the drag coefficient of a gas bubble moving in a liquid; σ is the surface tension coefficient; ρ and ρ' are the density of the liquid and the gas, respectively. For air and water $k_f = 0.5$, $\sigma = 7.35 \cdot 10^{-2}$ N/m, $\rho' = 1$ kg/m^3, $\rho = 10^3$ kg/m^3. U^2 is the mean-square velocity difference over a distance of the bubble diameter. In essence

$$U^2 = \langle \Delta U_L^2(2a_{cr}) \rangle, \tag{11}$$

where $\Delta U_L(x)$ is the velocity difference between the points spaced at a distance x in a turbulent flow. From formula (11) it is seen that in the case of a turbulent isotropic velocity field U^2 may be evaluated as the longitudinal structural function $D_{LL}(r, t)$ calculated at $r = 2a_{cr}$ [3]:

$$U^2 = D_{LL}(r, t)|_{r=2a_{cr}}. \tag{12}$$

Henceforth it is useful to employ the expression relating the structural function $D_{LL}(r, t)$ to the function $P_t(r)$ that describes the energy distribution of turbulent pulsations over length scales [4]:

$$D_{LL}(r, t) = 2\int_0^r P_t(r)\,dr. \tag{13}$$

Then the expression for U^2 in formula (10) in terms of $P_t(r)$ is as follows:

$$U^2 = 2\int_0^{2a_{cr}} P_t(r)\,dr. \tag{14}$$

With account for (14) formula (10) may be written as

$$a_{cr}(t)\int_0^{2a_{cr}(t)} P_t(r)\,dr = (1/k_f)^{1/3}(\sigma/\rho)(\rho/\rho')^{1/3}. \tag{15}$$

Solving Eq. (15), $a_{cr}(t)$ may be calculated as a function of time. We shall now write the breakup condition:

$$r > a_{cr}(t). \tag{16}$$

It is noteworthy that the breakup condition presented in the form of formulas (15) and (16) is an extension of formula (10) to the nonequilibrium case where a turbulent velocity field may be nonequilibrium, e.g., in the case of turbulence immediately behind a grid or in a zone of sudden expansion. In this case, $P_t(r)$ may be calculated from the closed equation of [4] for this function.

3. Derivation of the Equation for $f_t(r)$. We shall write an equation for the probability distribution function $f_t(r)$ in the case where bubbles break down into smaller ones without coalescence in a turbulent isotropic field.

We subdivide the entire size spectrum of the bubbles into macro- and microcomponents. The portion of the bubbles with radii less than r will be termed the microcomponent, and that with radii larger than r, the macrocomponent:

$$\int_0^r f_t(R)\,dR = M_u, \quad \int_r^\infty f_t(R)\,dR = M_a. \tag{17}$$

Next, we write the equality

$$\frac{\partial}{\partial t}\int_r^\infty f_t(R)\,dR = -W(r, t). \tag{18}$$

where $W(r, t)$ is the probability flux through the point r in the space of bubble sizes.

From the form of (18) and the fact that only the bubble breakup process is accounted for it follows that the function $W(r, t)$ is positive. We now assume that the structure of the probability flux $W(r, t)$ is related to $f_t(r)$ by the following formula:

$$W(r, t) = \int_r^\infty f_t(r)\frac{\hat{\omega}(r, R)}{\tau(r, R)}\,dR. \tag{19}$$

Here $\hat{\omega}(r, R)$ is the probability of transfer of the macrocomponent M_a to the microcomponent M_i through the point r. The function $\tau(r, R)$ is the time of breakup of a bubble of radius R into bubbles of radii less than R. We also assume that the breakup time does not depend on what bubbles the bubble of radius R splits into but depends only on the size of the splitting bubble:

$$\tau(r, R) \equiv \tau(R). \tag{20}$$

The time of bubble collapse in a deforming velocity field may be evaluated from a stability analysis of a liquid sphere in a flow of another liquid [5, 6]. This evaluation results in the formula

$$\tau(R) = (4n\sigma/\rho')(\rho'/\rho)^{1/2}\left(\int_0^{2R} P_t(r)\,dr\right)^{3/2}, \tag{21}$$

where

$$n = \ln(2R/\delta), \quad \delta = \left(\int_0^{2R} rP_t(r)\,dr\right)\Big/\left(\int_0^{2R} P_t(r)\,dr\right). \tag{22}$$

We represent $\hat{\omega}(r, R)$ in the form

$$\hat{\omega}(r, R) = \int_0^r \omega(\tilde{r}, R)\,d\tilde{r}. \tag{23}$$

Here the function $\omega(r, R)$ is the contribution of passage from the point R to the microcomponent $(0-r)$ through the boundary r to the total probability. Equality (23) merely indicates that the probability of breakup of a bubble of radius R into bubbles of radii $\tilde{r} < r$ consists of the sum of the probabilities of all the transitions $R \to \tilde{r}$ where $\tilde{r} \in (0-r)$. From the physical meaning sense of the function $\omega(\tilde{r}, R)$, it follows that the following equality is valid:

$$\int_0^R \omega(\tilde{r}, R)\,d\tilde{r} = \begin{cases} 1, & \text{if} \quad R > a_{cr}, \\ 0, & \text{if} \quad R < a_{cr}. \end{cases} \tag{24}$$

For the function $\hat{\omega}(r, R)$ this means the following condition:

$$\hat{\omega}(R, R) = \theta(R - a_{cr}). \tag{25}$$

Here $\theta(x)$ is the Heaviside step function.

Equalities (24) and (25) indicate that the probability for a bubble of radius R larger than the critical one to split into bubbles of any sizes smaller than R is equal to unity. If an initial bubble is too small to split in a prescribed turbulent velocity field, then $\hat{\omega}(R, R) = 0$.

Substituting expression (23) for $\hat{\omega}(r, R)$ into equality (19), we rewrite Eq. (18) in the form

$$\frac{\partial}{\partial t}\int_r^\infty f_t(R)\,dR = -\int_r^\infty \frac{f_t(R)}{\tau(R)}\left(\int_0^r \omega(\tilde{r}, R)\,d\tilde{r}\right)dR. \tag{26}$$

Differentiating the left- and right-hand sides of (26) with respect to r, we arrive at

$$\frac{\partial f_t(r)}{\partial t} = -\frac{f_t(r)}{\tau(r)}\int_0^r \omega(\tilde{r}, r)\,d\tilde{r} + \int_r^\infty \frac{f_t(R)}{\tau(R)}\,\omega(r, R)\,dR. \tag{27}$$

The first term on the right-hand side of equality (27) is the decrease in the probability of a bubble of radius r due to breakup into smaller ones of radius $\tilde{r} < r$. Using (23)-(25), we may simplify this term. As a result, Eq. (27) for $f_t(r)$ acquires the form

$$\frac{\partial f_t(r)}{\partial t} = -\frac{f_t(r)}{\tau(r)}\,\theta(r - a_{cr}(t)) + \int\limits_r^{\infty} \frac{f_t(R)}{\tau(R)}\,\omega(r, R)\,dR\,. \tag{28}$$

As inspection of (28) reveals, the first term on the right-hand side of the equation is the rate of decrease of the probability $f_t(r)$ at the point r, depending on the breakup time $\tau(r)$. This rate vanishes for bubbles of radius $r < a_{cr}(t)$.

The second term in the right side of Eq. (28) for $f_t(r)$ is the increase in the probability of bubbles of radius r due to breakup of all bubbles of radii $R > r$. As is obvious from physical considerations, the function $\omega(r, R)$ must satisfy the following condition:

$$\omega(r, R) = \begin{cases} \tilde{\omega}(r, R), & \text{if} \quad R > r,\ R > a_{cr}, \\ 0, & \text{if} \quad R < r,\ R < a_{cr}. \end{cases} \tag{29}$$

This equality may be written in the form

$$\tilde{\omega}(r, R) = \theta(R - r)\,\theta(r - a_{cr})\,\tilde{\omega}(r, R)\,. \tag{30}$$

The form of the function $\tilde{\omega}(r, R)$ should be chosen from physical suppositions. As is known from experiments, breakup of bubbles into smaller ones, if it occurs, may be implemented by numerous methods [2, 6]; however, quantitative information sufficient for constructing the function $\tilde{\omega}(r, R)$ is not available. Taking into consideration the qualitative experimental finding that in regimes with a Weber number differing slightly from its critical value a bubble breaks up into two large fragments [6], the hypothesis that the function $\tilde{\omega}(r, R)$ describes bubble breakup mainly into two equal parts may be adopted. Although other possibilities are not ruled out, they decrease with increase in the asymmetry of bubble breakup in accordance with the normal distribution law.

The segment of the boundary of the domain of nonzero values of the function $\omega(r, R)$ that is described by the equation $R = a_{cr}(t)$ in accordance with formula (15) is displaced as the structure of the turbulent velocity field changes in a liquid that is described by the function $P_t(r)$. The maximum R values with respect to which the integration in (28) is performed are not restricted by $\omega(r, R)$. This restriction is imposed by the initial probability distribution $f_0(R)$ and the circumstance that no bubbles larger than those prescribed initially appear in the system during turbulent breakup.

Bubbles that split into two equal parts, i.e., with $R = 2^{1/3}r$, give the maximum contribution to the rate of decrease of the function $f_t(r)$ at the point r as a result of breakup of bubbles of radius R into two parts. Therefore $\tilde{\omega}(r, R)$ acquires the following form:

$$\tilde{\omega}(r, R) = \frac{1}{N}\exp\left\{ -\left(r - R/\sqrt[3]{2}\,\right)^2/2\sigma_r^2 \right\}. \tag{31}$$

The value of the dispersion σ_r is chosen sufficiently small for the function $\omega(r, R)$ to be small at the boundary of the domain of definition in the variable r ($r = 0$, $r = R$). If we use the $3\sigma_r$ rule, then we obtain $\sigma_r \le 0.06R$. In (31), N is a normalization factor. It may be calculated from the condition

$$\int\limits_0^R \tilde{\omega}(r, R)\,dr = 1\,, \tag{32}$$

which states the obvious fact that creation of a bubble of radius $r < R$ due to turbulent breakup of a bubble of radius $R > a_{cr}(t)$ is certain.

Integration of (31) in accordance with (32) yields

$$N = N(R, \sigma_r) = \sigma_r\sqrt{\pi/2}\,\left\{ \Phi\left[(\sqrt[3]{2} - 1)\,R/(2^{5/6}\sigma_r)\right] + \Phi\left[R/(2^{5/6}\sigma_r)\right] \right\}, \tag{33}$$

where

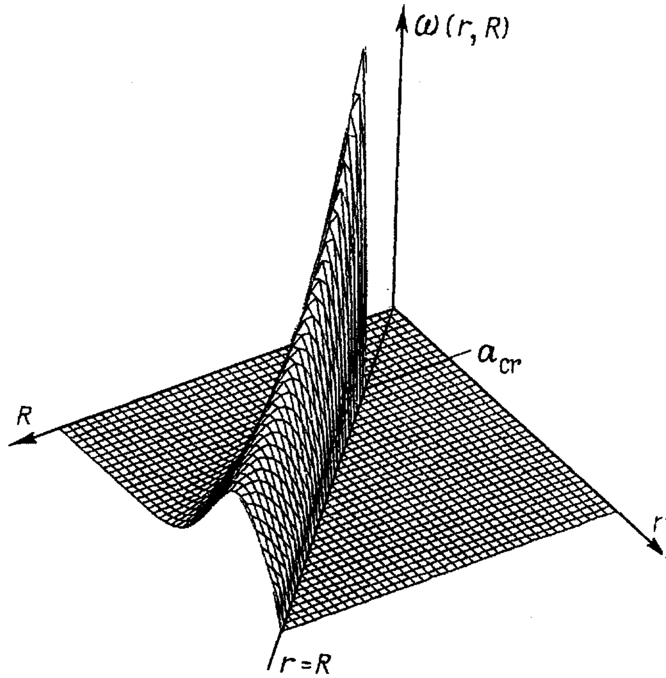


Fig. 1. Form of the function $\omega(r, R)$ determined by formulas (31)-(34).

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)\, dt. \tag{34}$$

The function $\omega(r, R)$ defined on the plane (r, R) is shown in Fig. 1.

It should be noted that in solving Eq. (28) for $f_t(r)$ at points $r < a_{cr}(t)$ the first term on the right-hand side of the equation disappears, and integration in the second term, as seen from the graph of $\omega(r, R)$, starts from the point $R = a_{cr}(t)$.

From the definition of $f_t(r)$ as a probability density it follows that the normalization condition (1) must be fulfilled for it at any moment of time. If at the initial moment of time the function is stipulated to be normalized to unity, then to preserve normalization at any t it is necessary to fulfill the condition

$$\frac{\partial}{\partial t} \int_0^\infty f_t(r)\, dr = 0. \tag{35}$$

This condition is actually fulfilled, which is easily proved by integrating the right-hand side of Eq. (28).

Thus, the equation for $f_t(r)$ in the form of (28) together with formulas (21), (22) for $\tau(R)$ and (30), (31), (33), (34) for $\omega(r, R)$ and the closed equation for $P_t(r)$ that describes the energy distribution of turbulent fluctuations over different scales [4] constitute a closed system of equations. Solving this system, one may follow the evolution of the probability distribution of bubbles sizes in an isotropic damping turbulent flow. The initial form of $f_t(r)$ may be prescribed as a slightly smeared one-scale distribution with the mean length scale r_0. As will be shown later, the results of numerical calculations are not very sensitive to the form of the initial distribution.

4. System of Equations in Dimensionless Form and Numerical Solution. Prior to solving the system of equations, we shall choose characteristic quantities and write the complete system of equations in the dimensionless form. As the characteristic quantities we choose:

- the length scale $r_x = L$, where L is the mean scale of the initial vortex;
- the value of $f_t(r) \Rightarrow f_x = 1/L$;
- the value of $\omega(r, R) \Rightarrow \omega_x = 1/L$;
- the velocity of turbulent pulsations $U'_x = \sqrt{B(0)}$, where $B(0)$ is the mean-square velocity of longitudinal pulsations at the initial moment of time;
- the time $t_x = r_x/U'_x = L/\sqrt{B(0)}$;

- the function $P_t(r) \Rightarrow P_x = B(0)/L$.

Denoting dimensionless variables by symbols with a bar, we obtain

$$f_t(r) = \bar{f}_t(\bar{r})/L; \quad \omega(r, R) = \omega(\bar{r}, \bar{R})/L;$$

$$\tau(R) = \bar{\tau}(\bar{R})\, L/\sqrt{B(0)}; \tag{36}$$

$$\partial/\partial t = \left(\sqrt{B(0)}/L\right) \partial/\partial \bar{t}.$$

Here $\bar{r} = r/L$; $\bar{R} = R/L$.

The equation for $f_t(r)$ in terms of dimensionless variables looks like (28):

$$\partial \bar{f}_{\bar{t}}(\bar{r})/\partial \bar{t} = \bar{f}_{\bar{t}}(\bar{r})\, \theta(\bar{r} - \bar{a}_{cr}(\bar{t}))/\bar{\tau}(\bar{r}) + \int_{r}^{\infty} \bar{f}_{\bar{t}}(\bar{r})\, \bar{\omega}(\bar{r}, \bar{R})\, d\bar{R}/\bar{\tau}(\bar{R}). \tag{37}$$

The expression for $\tau(r)$ acquires the form

$$\tau(R) = \frac{8n\,(\rho'/\rho)^{1/2} r_0}{L\,\mathrm{We}_0} \left\{ \int_0^{2\bar{R}} \bar{P}_t(\bar{r})\, d\bar{r} \right\}^{-3/2}, \tag{38}$$

where We_0 is the Weber number determined in terms of the initial bubble radius r_0 and the mean-square velocity $B(0)$:

$$\mathrm{We}_0 = 2r_0 \rho' B(0)/\sigma, \tag{39}$$

$$n = \ln(2\bar{R}/\delta): \ \delta = \int_0^{2\bar{R}} \bar{r} \bar{P}_{\bar{t}}(\bar{r})\, d\bar{r} \Big/ \int_0^{2\bar{R}} \bar{P}_{\bar{t}}(\bar{r})\, d\bar{r}. \tag{40}$$

The equality for determination of $\bar{a}_{cr}(\bar{t})$ becomes

$$\bar{a}_{cr}(\bar{t}) \int_0^{2\bar{a}_{cr}(\bar{t})} \bar{P}_{\bar{t}}(\bar{r})\, d\bar{r} = \frac{2r_0}{L\,\mathrm{We}_0}\, (1 - k_f)^{1/3} (\rho'/\rho)^{2/3}. \tag{41}$$

The form of $\bar{\omega}(\bar{r}, \bar{R})$ in terms of dimensionless variables remains unchanged:

$$\omega(\bar{r}, \bar{R}) = \theta(\bar{R} - \bar{r})\, \theta(\bar{R} - \bar{a}_{cr}(\bar{t}))\, \exp\left(-(\bar{r} - \bar{R}/\sqrt[3]{2})^2/2\bar{\sigma}_r^2\right)/\bar{N}, \tag{42}$$

where

$$\bar{N} = \bar{\sigma}_r \sqrt{\pi/2} \left\{ \Phi\left(\frac{\bar{R}}{\bar{\sigma}_r} \frac{\sqrt{2} - 1}{2^{5/6}} \right) + \Phi\left(\frac{\bar{R}}{\bar{\sigma}_r} \frac{1}{2^{5/6}} \right) \right\}. \tag{43}$$

The initial condition for $\bar{f}_{\bar{t}}(\bar{r})$ may be written in the parabolic form $\bar{f}_0(r) = A(\bar{r} - \bar{r}_0)^2 + B$ where $\bar{r}_0 = r_0/L$. If we choose the halfwidth of the initial probability distribution of bubble radii to be equal to Δ, then the normalization condition for $\bar{f}_0(\bar{r})$ yields the equalities $A = -3/4\Delta^3$, $B = 3/4\Delta$. Thus

$$\bar{f}_0(r) = -3(\bar{r} - \bar{r}_0)^2/4\bar{\Delta}^3 + 3/4\bar{\Delta}, \tag{44}$$

where $\bar{\Delta} = \Delta/L$.

The equation for $\bar{P}_{\bar{t}}(\bar{r})$ has the form

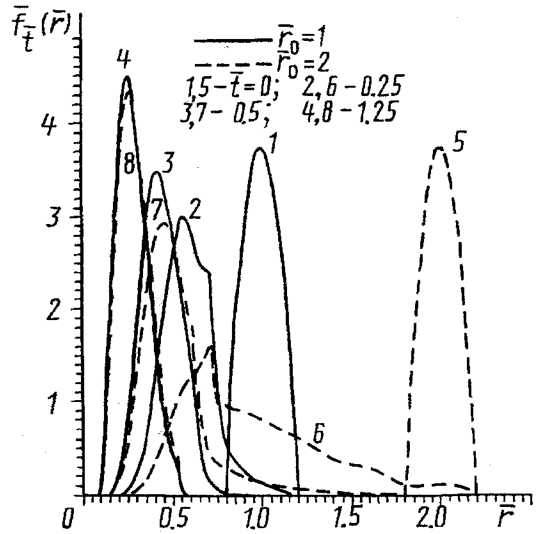


Fig. 2. Evolution of the function $\bar{f}_{\bar{t}}(\bar{r})$ for different initial mean radii $\bar{r}_0$ of the bubbles ($\mathrm{Re}_0 = 10^4$, $\bar{\Delta} = 0.2$, $t = \bar{t}\cdot 10^{-4}$ sec, $r_0 = \bar{r}_0\cdot 10^{-3}$ m).

$$\partial \bar{P}_{\bar{t}}(\bar{r})/\partial \bar{t} = \partial \left\{ \left(2/\mathrm{Re}_0 + 2\gamma \int_0^r \sqrt{\bar{\rho}\,\bar{P}_{\bar{t}}(\bar{\rho})}\, d\bar{\rho} \right) (\partial/\partial \bar{r} + 4/\bar{r})\, \bar{P}_{\bar{t}}(\bar{r}) \right\} \Big/ \partial \bar{r}\,, \tag{45}$$

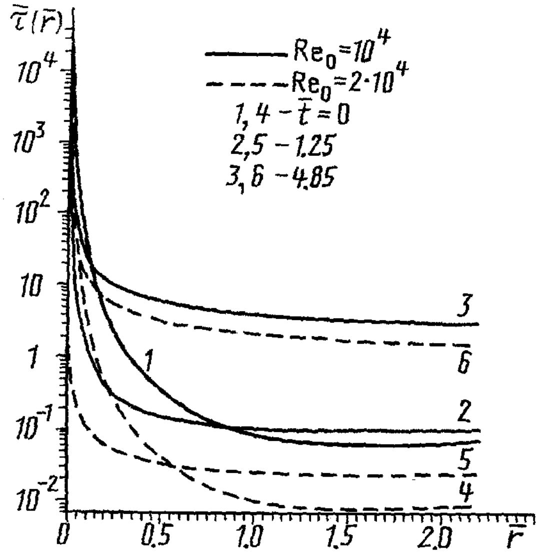
where $\gamma = 0.24$. The initial and boundary conditions for $\bar{P}_{\bar{t}}(\bar{r})$ are as follows:

$$\bar{P}_{\bar{t}}(\bar{r})\,|_{\bar{t}=0} = 2\bar{r}\exp(-\bar{r}^2)\,; \quad P_{\bar{t}}(\bar{r})\,|_{r=0} = P_{\bar{t}}(\bar{r})\,|_{r=\infty} = 0\,. \tag{46}$$

Equations (37), (45) were solved by an implicit finite-difference method combined with iterations of nonlinearities. The initial condition for $\bar{P}_{\bar{t}}(\bar{r})$ was prescribed by (44). The initial and boundary conditions for $\bar{P}_{\bar{t}}(r)$ were given by (46). In solving for $\bar{P}_{\bar{t}}(\bar{r})$, the factorization method was employed. The step of the spatial grid was chosen to increase with $\bar{r}$. The convergence of the iterations at each time step was evaluated by fulfilment of the inequalities $|(P_s - P_{s+1}/(P_s + P_{s+1})| < 10^{-5}$, $|(f_s - f_{s+1})/(f_s + f_{s+1})| < 10^{-4}$, where P_s and f_s are the values of the functions $\bar{P}_{\bar{t}}(\bar{r})$ and $\bar{f}_{\bar{t}}(\bar{r})$ in the iteration with numbered by s.

5. Results of Numerical Solution of the System of Equations for $f_t(r)$. Figure 2 shows the evolution of the probability distribution function of bubble radii $\bar{f}_{\bar{t}}(\bar{r})$ for different initial mean radii of bubbles. The initial distribution $\bar{f}_0(\bar{r})$ is given in accordance with formula (44) for $\bar{r}_0 = 1$ and $\bar{r}_0 = 2$. As is seen, the evolution of $\bar{f}_{\bar{t}}(\bar{r})$ under the action of the turbulent velocity field is manifested mainly in a shift toward smaller bubble sizes. In a time $t = 1.25\cdot 10^{-4}$ sec the distribution function is seen to attain its stationary value. A comparison of the evolution of $\bar{f}_{\bar{t}}(\bar{r})$ for two initial mean bubble radii r_0 allows the conclusion that this parameter exerts a weak influence on the time for attaining the final distribution. Inspection of the evolution of $\bar{f}_{\bar{t}}(\bar{r})$ in the cases when the mean bubble radius is twice the mean turbulence scale and when they coincide (Figs. 5 and 1, respectively) reveals that the corresponding distributions almost coincide in shape already at $t = 0.5\cdot 10^{-4}$ sec. This insensitivity of the total breakup time to a change in the initial scale is due to the fact that the breakup time of a bubble of radius r decreases substantially with increase in the radius. As is seen from Fig. 3, the breakup time $\tau(r)$ increases drastically when the radius is smaller than $\bar{r} = 0.5$. As a result, the final almost stationary probability distribution is established slowly. Moreover, $\tau(r)$ increases greatly as the turbulent energy decays. Divergence of the curves for different Re_0 indicates that $\tau(r)$ decreases with increase in the Reynolds number.

Figure 4 shows the critical bubble radius $a_{\mathrm{cr}}(t)$ as the turbulent flow evolves for different Re. First the critical radius is seen to decrease due to the size reduction of the turbulent flow. Then $a_{\mathrm{cr}}(t)$ begins to grow, which is associated with attenuation of the turbulent energy. A comparison of the change in the $a_{\mathrm{cr}}(t)$ curves with the evolution of $f_t(r)$ demonstrates that all the bubbles with radii $r > a_{\mathrm{cr}}(t)$ manage to break up before the critical radius begins to increase due to damping of the turbulence.



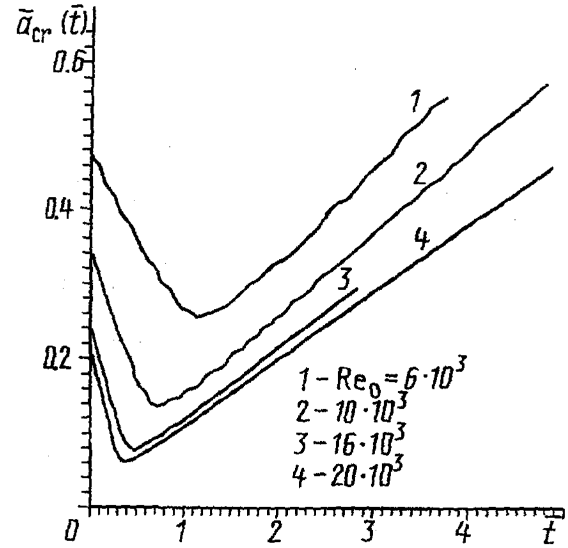


Fig. 3. Characteristic breakup time of bubbles versus radius for different initial Reynolds numbers.

Fig. 4. Time variation of the critical bubble radius for different initial Reynolds numbers.

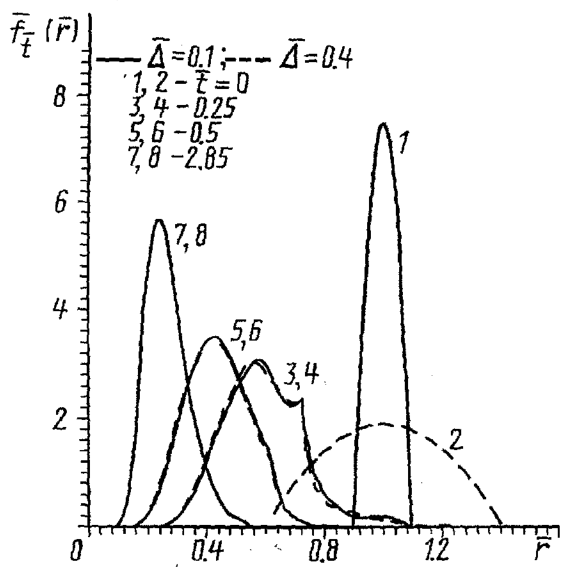


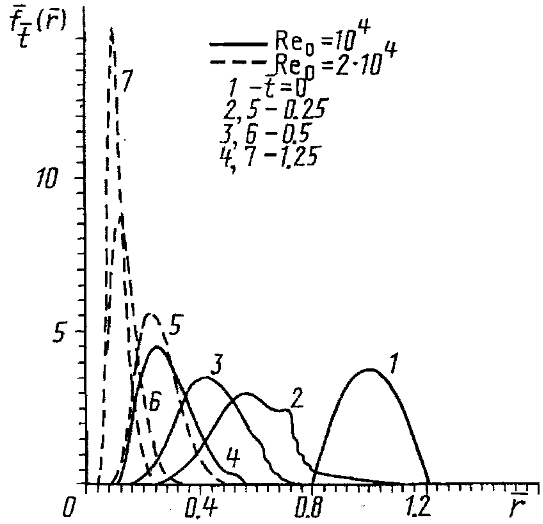
Fig. 5. Evolution of the function $\bar{f}_{\bar{t}}(\bar{r})$ for different $\bar{\Delta}$ values in the initial distribution ($Re_0 = 10^4$, $t = \bar{t}\cdot 10^{-4}$ sec, $\bar{r}_0 = 1$).

The aforementioned insensitivity of the total evolution time for $\bar{f}_{\bar{t}}(\bar{r})$ to the form of the initial distribution is seen in Fig. 5, where the evolution of this function is shown for two different values of the parameter Δ which determines the dispersion of the initial distribution at one and the same mean bubble radius. Already at $t = 0.5\cdot 10^{-4}$ sec the functions almost coincide , although their forms differ strongly at $t = 0$. Upon further evolution this coincidence is preserved.

Thus, the mean initial bubble size and the dispersion of the initial distribution prove to be insignificant for control of the resultant size distribution of the bubbles. This conclusion is confirmed by investigation of integral characteristics of $\bar{f}_{\bar{t}}(\bar{r})$.

The mean bubble radius is determined from the relation

$$\bar{r}(\bar{t}) = \int_0^\infty \bar{r}\bar{f}_{\bar{t}}(\bar{r})\,d\bar{r}. \tag{47}$$



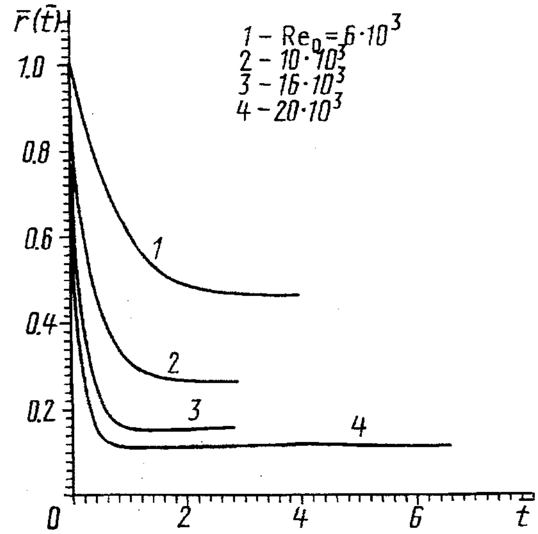


Fig. 6. Evolution of the function $\bar{f}_{\bar{t}}(\bar{r})$ for different Re_0 values ($L = 10^{-3}$ m, $\bar{\Delta} = 0.2$, $t_x = 10^{-4}$ sec).

Fig. 7. Time variation of the mean bubble radius for different initial Re_0 in a damping turbulent flow ($L = 10^{-3}$ m, $\bar{\Delta} = 0.2$, $t_x = 10^{-4}$ sec, $\bar{r}_0 = 1$).

Calculations show that for any initial values the mean radius $\bar{r}(\bar{t})$ tends to one and the same value, approximately equal to $\bar{r}(\infty) = 0.3$. Here, the factor of proportion between the initial mean length scale of the turbulent velocity field and the initial mean size of the bubbles is insignificant until the initial radius exceeds the critical bubble size determined by formula (41) at any moment of time. As has been shown, the dependence of $\bar{r}(\bar{t})$ on the dispersion of the initial distribution is insensitive to the choice of this quantity.

The dispersion of the distribution $\bar{f}_{\bar{t}}(\bar{r})$ as a function of time is determined from the formula

$$\sigma(t) = \sqrt{\left(\overline{r^2(t)} - \bar{r}^2(t)\right)}\,, \tag{48}$$

where $\bar{r}(t)$ is determined by (47) and $\overline{r^2(t)}$ is calculated from the expression

$$\overline{r^2(t)} = \int_0^\infty r^2 \bar{f}_{\bar{t}}(\bar{r})\, d\bar{r}\,. \tag{49}$$

The dispersion $\sigma(t)$ of the distribution undergoes a complicated evolution but it ultimately attains one and the same value for different $\bar{r}_0$. The jump in the dispersion $\sigma(t)$ in the initial stage of evolution is associated with spectrum rearrangement due to transfer of the probability density from the region of large radii to that of small ones. The fact that the dispersion attains a constant value at $\bar{t} = 2.5$ points to cessation of evolution of $\bar{f}_{\bar{t}}(\bar{r})$. This means that the system has no bubbles whose radii are larger than the critical one, i.e., bubbles capable of breaking up.

The change in the dispersion $\bar{\sigma}(\bar{t})$ for different $\bar{\Delta}$ values is the same in the initial form of the distribution as that in the dispersion for different $\bar{r}_0$ values. In the initial stage of distribution evolution a sudden increase in the dispersion is observed, followed by a gradual decrease in $\bar{\sigma}(\bar{t})$ with attainment of a constant value as the bubbles cease to break up.

From the aforesaid it follows that a change in the form of the initial distribution $\bar{f}_0(\bar{r})$ does not entail a substantial change in $\bar{f}_{\bar{t}}(\bar{r})$ and does not exert a pronounced influence on the time for attaining a stationary distribution.

A more effective means of controlling of the probability distributions of bubble sizes is a change in the turbulence parameters. Figure 6 shows the evolution of the function $\bar{f}_{\bar{t}}(\bar{r})$ calculated for different Re_0. Variation of Re_0 was achieved with a constant length scale L by changing the energy of turbulent pulsations. It is seen that bubbles break up more vigorously at greater Re_0, which agrees with the aforementioned decrease in the breakup

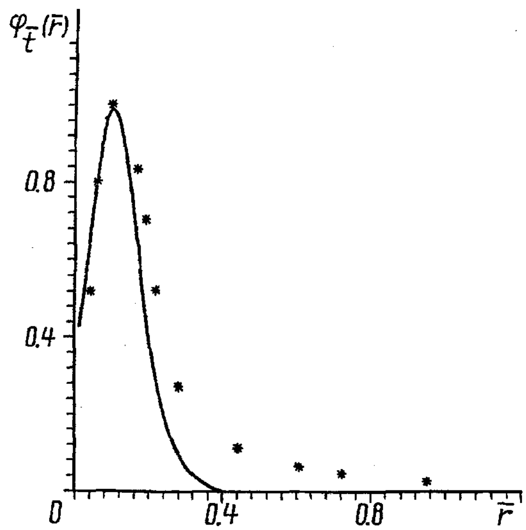


Fig. 8. Comparison of the theoretical (curve) ($Re_0 = 3 \cdot 10^4$, $L = 1.5$ mm, $\bar t = 3.015$) and experimental (points) distributions $\varphi_{\bar t}(\bar r)$.

time $\tau(R)$ with increase in Re_0 (see Fig. 3). In this case, the probability distribution $\bar f_{\bar t}(\bar r)$ for higher Re_0 numbers is stabilized at smaller bubble sizes.

As is seen from Fig. 7, where the change in the mean bubble radius with time is shown for different Re_0 and a constant initial length scale, the distribution is stabilized more quickly at large Re_0. At $Re_0 = 2 \cdot 10^4$ the mean bubble radius is smaller by a factor of 2.5 than at $Re_0 = 10^4$.

The results in Figs. 6, 7 demonstrate that an increase in the turbulent energy is an effective means of controlling the size distribution of bubbles. As is seen from Fig. 6, the final distribution is narrower at large Re_0, i.e., the dispersion of the bubble size distribution decreases substantially with increase in the turbulent energy of the flow.

Variation of the mean bubble size in a stationary distribution may be also achieved by changing the initial scale of the turbulent velocity field. Calculation results for the time variation of the mean bubble radius and the mean dispersion $\sigma(t)$ for different initial scales L and a fixed Re_0 show that the smaller the scale of the initial structural of the turbulent velocity field, the more vigorous the bubble breakup and the more rapid the stabilization.

Examination of the final mean bubble radius versus the initial macroscale of the velocity field shows that a twofold decrease in L results in a 2.5-fold decrease in $\bar r$. Thus, by changing the initial scale of the turbulence length one may control effectively the final size distribution of the bubbles.

Figure 8 shows the curve $\varphi_{\bar t}(\bar r) = \bar f_{\bar t}(r)/\bar f_{\bar t}(\bar r)_{max}$ calculated theoretically using the proposed model and experimental points obtained at the outlet of a cavitation generator. It is seen that theory and experiment agree satisfactorily for small bubble sizes. The fact that the probability of the experimental dependence exceeds somewhat that of the theoretical one for large bubble sizes is explained by neglect of coalescence in the present model, which may occur under the experimental conditions.

Conclusion. A theoretical model of bubble breakup in a turbulent isotropic damping flow of an incompressible liquid is proposed. It is assumed that the turbulence parameters for the liquid are independent of the presence of bubbles in the flow and they are calculated from a closed equation for the function $P_t(r)$ that describes the distribution of turbulent velocity pulsations over different length scales. The bubble size distribution density is calculated from a linear integral equation whose variable coefficients are linearly related to the function $P_t(r)$. Having solved numerically the proposed model, we draw the following conclusions.

The evolution of the probability distribution density of bubble radii $f_t(r)$ under the action of a turbulent velocity field consists mainly in a shift toward small bubble radii in a finite time.

The breakup time of bubbles and the final form of $f_t(r)$ are not very sensitive to the mean scale and dispersion of the initial distribution of bubbles.

The rate of bubble breakup is such that all the bubbles manage to break up before the critical radius increases due to dissipation of the turbulent energy.

The mean radius and dispersion of the final distribution of bubbles are independent of the corresponding parameters of the initial distribution.

The sudden rise in dispersion for intermediate times is explained by transfer of probability from the region of large-radius bubbles to the region of small-radius bubbles.

With increase in the initial Reynolds number with a fixed initial scale of the turbulent velocity field the bubbles break up more vigorously. The distribution stabilizes more quickly at large Re_0.

At large Re_0, $f_t(r)$ stabilizes at smaller values of the mean radius and dispersion.

A decrease in the initial length scale of the turbulent velocity field at a fixed initial Reynolds number yields results similar to those obtained for an increase in Re_0 with a fixed L.

The authors thank the Foundation for Fundamental Investigations of Belarus for financing the work.

NOTATION

$f_t(r)$, probability density of the bubble radius; $P_t(r)$, turbulence energy distribution over different length scales; a_{cr}, critical bubble radius; We_0, Weber number; $\omega(r, R)$, function describing the breakup of bubbles of radius R into bubbles of radius r; σ, surface tension coefficient; $\tau(R)$, characteristic time of bubble breakup; Re_0, turbulent Reynolds number determined in terms of the initial macroscale L of the turbulent velocity field and the initial energy of the turbulent flow $B(0)$; $\bar{r}(t)$, mean bubble radius as a function of time; σ_r, dispersion of the distribution $\omega(r, R)$; Δ, width parameter of the initial distribution $f_0(r)$.

REFERENCES

1. B. I. Puris, G. Sh. Klyumel, V. A. Gertsovich, and É. P. Polesskii, Transport Processes in Turbulent Flows [in Russian], Minsk (1988), pp. 123-137.
2. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Moscow (1959).
3. A. S. Monin and A. M. Yaglom, Statistical Hydromechanics [in Russian], Pt. 2, Moscow (1967).
4. V. A. Sosinovich and V. A. Tsyganov, Inzh.-Fiz. Zh., **46**, No. 2, 219-225 (1984).
5. G. Birkhoff, Hydrodynamics [Russian translation], Moscow (1963).
6. R. I. Nigmatulin, Dynamics of Multiphase Media [in Russian], Pt. 1, Moscow (1987).